Land Area Determination

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Outline

I. Introduction
II. Methods of Determining Land Area
III. Rules in Computing DMD
IV. Sample Problem and Solution
Introduction

- Area determination is one of the primary objectives of land surveys.
- Area is a final determinant of the total market value of a piece of land put on sale to the public.
Methods of Determining Land Area

1. Area by Triangles

The area can be found by dividing the lot into a series of triangles, making the necessary measurements and then calculating the area by any of the usual trigonometric formulas.
Area by Triangles

Area by Triangles Formula

- \[ A = \frac{1}{2} a \cdot b \sin C \]

- \[ A = \sqrt{s(s-a)(s-b)(s-c)} \]

where \( s = \frac{1}{2}(a+b+c) \)
Methods of Determining Land Area

2. Area by Coordinate Squares

The scaled drawing is marked off in squares of unit areas then counted. This is used to approximate areas.
Coordinates Squares Illustration
Methods of Determining Land Area

3. Area by Polar Planimeter

The polar planimeter is a mechanical device used to determine the area of any shape of figure plotted to a known scale by directly tracing the perimeter and reading the result from the scale.
Parts of a Polar Planimeter

- Carriage
- Measuring Wheel
- Pole Arm
- Pole Weight
- Tracer Arm
Sample Polar Planimeters
Methods of Determining Land Area

4. Area by Offsets from Straight Lines

Areas with irregular or curved boundaries are usually measured by establishing a base line conveniently near and by taking offsets at regular intervals from the base line to the boundary.

4.1 Trapezoidal Rule
4.2 Simpson’s One-Third Rule
Trapezoidal Rule Illustration

\[ h_0 = 35 \quad h_1 = 25 \quad h_2 = 30 \quad h_3 = 40 \quad h_n = 10 \]
Illustrative Problem

1. **TRAPEZOIDAL RULE**: A series of perpendicular offsets were taken from a transit line to an irregular boundary. These offsets were taken 2.5 meters apart and were measured in the following order: 0.0, 2.6, 4.2, 4.4, 3.8, 2.5, 4.5, 5.2, 1.6, and 5.0 meters. By the trapezoidal rule find the area included between the transit line, the curved boundary, and the end offsets.

\[
d = 2.5 \text{ meters (common interval between offsets)}
\]

\[
\text{AREA} = d \left( \frac{h_1 + h_{10}}{2} + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9 \right)
\]

\[
= 2.5 \left( \frac{0.0 + 5.0}{2} + 2.6 + 4.2 + 4.4 + 3.8 + 2.5 + 4.5 + 5.2 + 1.6 \right) = 78.25 \text{ sq m}
\]
Illustrative Problem

2. **SIMPSON'S ONE-THIRD RULE**: From a transit line to the edge of a river, a series of perpendicular offsets are taken. These offsets are spaced 4.0 meters apart and were measured in the following order: 0.5, 1.4, 2.5, 5.6, 8.5, 7.4, 3.8, 5.1, and 2.3 meters. By Simpson's One-Third Rule, compute the area included between the transit line, the river's edge, and the end offsets.

\[ a = 4.0 \text{ meters} \]
(common interval between offsets)

\[ \text{AREA} = \frac{d}{3} \left[ h_1 + h_9 + 2(h_3 + h_5 + h_7) + 4(h_2 + h_4 + h_6 + h_8) \right] \]

\[ = \frac{4}{3} \left[ 0.5 + 2.3 + 2(2.5 + 8.5 + 3.8) + 4(1.4 + 5.6 + 7.4 + 5.1) \right] = 147.20 \text{ sq m} \]
Methods of Determining Land Area

5. Area by Coordinates

The method of coordinates is based on the following rule in analytic geometry: If the vertices of the figure are taken in order around the figure, the area is equal to one-half the sum of the products of each ordinate multiplied by the difference between the two adjacent abscissas always subtracting the preceding from the following abscissa.
Area Calculation by Coordinate Method

**FROM THE GIVEN FIGURE:**

\[
\text{Area } ABCD = \text{Area Trapezoid } (aABb) + \text{Area Trapezoid } (bBCc) - \text{Area Trapezoid } (aADD) - \text{Area Trapezoid } (dDCc)
\]

**IN TERMS OF X AND Y COORDINATES:**

\[
\text{Area } ABCD = \frac{1}{2} (X_1 + X_2)(Y_1 - Y_2) + \frac{1}{2} (X_2 + X_3)(Y_2 - Y_3) - \frac{1}{2} (X_1 + X_4)(Y_1 - Y_4) - \frac{1}{2} (X_4 + X_3)(Y_4 - Y_3)
\]

**EXPANDING THE EQUATION:**

\[
\text{Area } ABCD = \frac{1}{2} \left[ (X_1 Y_1 - X_1 Y_2 + X_2 Y_1 - X_2 Y_2 + X_2 Y_2 - X_2 Y_3 + X_3 Y_2 - X_3 Y_3) \\
(X_4 Y_1 - X_1 Y_4 + X_4 Y_1 - X_4 Y_4 + X_4 Y_4 - X_4 Y_3 + X_3 Y_4 - X_3 Y_3) \right]
\]
Area Calculation by Coordinate Method

Diagram showing the area calculation method with points A, B, C, and D defined by their coordinates.
SIMPLIFYING:

Area $ABCD = \frac{1}{2}(X_2 Y_1 - X_1 Y_2 + X_3 Y_2 - X_2 Y_3 + X_4 Y_3 - X_3 Y_4 + X_1 Y_4 - X_4 Y_1)$

The above formula is based on the summation of the area of a series of trapezoids. The coordinates used are the Northings and Eastings for the points or stations of the traverse. North and east coordinates are plus; south and west are minus.

The formula may also be expressed in determinant form, thus:

$$Area\ ABCD = \frac{1}{2} \begin{vmatrix} X_1 & X_2 & X_3 & X_4 & X_1 \\ Y_1 & Y_2 & Y_3 & Y_4 & Y_1 \end{vmatrix}$$

To be suitable for determining the area of a closed traverse of any number of sides, the formula may be expressed in a more general form as follows:

$$Area\ ABCD = \frac{1}{2} \begin{vmatrix} X_1 X_2 X_3 & \ldots & X_n & X_1 \\ Y_1 Y_2 Y_3 & \ldots & Y_n & Y_1 \end{vmatrix}$$

WHERE:

a) $X_1, X_2, \text{etc.}$ are the x-coordinates or Eastings
b) $Y_1, Y_2, \text{etc.}$ are the y-coordinates or Northings
c) $n$ is the last numbered station or point of the traverse.
Methods of Determining Land Area

6. Area by Double Meridian Distance Method

- **Meridian distance** – is the distance of the midpoint of a line to the reference meridian.
- This method is an adaptation of the coordinates method and is convenient to use when the latitudes and departures of the traverse are known.
DMD Illustration
The double meridian distance of any line is twice its meridian distance.

\[ AB = 2xGG' = GG' + KL \text{ or departure of } AB \]

\[ BC = 2xFF' = 2xGG' + 2xKL + 2xLF' \]
\[ = DMD \text{ of } AB + \text{dep of } AB + \text{dep of } BC \]

\[ CD = 2xHH' = 2xHM + 2xMN - 2xNH' \]
\[ = DMD \text{ of } BC + \text{dep of } BC + \text{dep of } CD \]
Rules in Computing DMD

1. The DMD of the first line is equal to the departure of that line.

2. The DMD of any other line is equal to the DMD of the preceding line, plus the departure of the preceding line, plus the departure of the line itself.

3. The DMD of the last line is numerically equal to the departure of the line but with opposite sign.
Sample Problem

Given the following data of a balanced closed traverse, determine the area by DMD Method.

<table>
<thead>
<tr>
<th>LINE</th>
<th>LAT</th>
<th>DEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>+ 10.49</td>
<td>+ 8.76</td>
</tr>
<tr>
<td>2-3</td>
<td>- 12.66</td>
<td>+ 11.30</td>
</tr>
<tr>
<td>3-4</td>
<td>- 15.23</td>
<td>+ 13.48</td>
</tr>
<tr>
<td>4-5</td>
<td>- 7.05</td>
<td>- 27.19</td>
</tr>
<tr>
<td>5-1</td>
<td>+ 24.45</td>
<td>- 6.35</td>
</tr>
</tbody>
</table>
Sample Problem Solution

<table>
<thead>
<tr>
<th>LINE</th>
<th>LAT</th>
<th>DEP</th>
<th>DMD</th>
<th>DA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>+ 10.49</td>
<td>+ 8.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>- 12.66</td>
<td>+ 11.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-4</td>
<td>- 15.23</td>
<td>+ 13.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-5</td>
<td>- 7.05</td>
<td>- 27.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-1</td>
<td>+ 24.45</td>
<td>- 6.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2A = 

\[ A = \Box \]
# Sample Problem Solution

<table>
<thead>
<tr>
<th>LINE</th>
<th>LAT</th>
<th>DEP</th>
<th>DMD</th>
<th>DA</th>
</tr>
</thead>
<tbody>
<tr>
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<td>+ 8.76</td>
<td>+ 8.76</td>
<td>+ 91.89</td>
</tr>
<tr>
<td>2-3</td>
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<td>+ 11.30</td>
<td>+ 28.82</td>
<td>- 364.86</td>
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<tr>
<td>3-4</td>
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<td>+ 53.60</td>
<td>- 816.33</td>
</tr>
<tr>
<td>4-5</td>
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<td>- 27.19</td>
<td>+ 39.89</td>
<td>- 281.22</td>
</tr>
<tr>
<td>5-1</td>
<td>+ 24.45</td>
<td>- 6.35</td>
<td>+ 6.35</td>
<td>+ 155.26</td>
</tr>
</tbody>
</table>

\[ 2A = -1215.26 \]

\[ A = 607.63 \text{ m}^2 \]
Gracias!